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Unified Treatment of Fermions and Bosons

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Dirac's matrices can be interpreted as an 8-rank covariant antisymmetric tensor field on an 11-dimensional manifold (space-time $\times S^7$) enforcing a linkage between the Lorentz transformation and "rotations" of S^7, conferring spinorial properties on any quantity having an index in the inner space S^7.

We present an alternative to supersymmetry (treating fermions and bosons on an equal footing) and preon theory (bosons as composites from fermions) by showing that fermions can also be viewed as special bosonic states. The model works in an 11-dimensional manifold, where each point of space-time (x^μ) is a microscopic S^7 (topology $\mathcal{R} \times S^7$).

The basic idea can best be explained by the simpler model of a cylindrical screw (topology $\mathcal{R} \times S^1$), as a model for a vacuum with spin structure, because a translation is only a symmetry if it is linked to a rotation of the internal space S^1. In the actual model a Lorentz transformation $\Lambda^\mu{}_\nu$ is linked to a deformation of S^7. If $\Lambda^\mu{}_\nu$ is a rotation by the angle α, it is linked to a rotation by $\alpha/2$ of S^7. Thus the spinors, as representations of the $SL(C, 2)$, come in. If $\Lambda^\mu{}_\nu$ is a boost, it is linked to a deformation of S^7 to an ellipsoid. Since S^7 is only a topological sphere (not a metrical sphere), this is a symmetry.

Only antisymmetrical tensor fields, Cartan's exterior derivative d, and the wedge product $\wedge$ (antisymmetrized tensor product) occur as elements of the theory. Field equations are generally covariant. The metric is not a fundamental 11-dimensional quantity as in Kaluza theories, but can be derived as a composite quantity for four-dimensional subspace (space-time). Spinors occur not for the 11-dimensional space, but only for space-time [representations of $SL(C, 2)$].

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Putting aside all mathematical technicalities, the model is based on the well-known invariance of Dirac's γ-matrices

$$\gamma^\mu = \Lambda^\mu{}_\nu s^{-1} \gamma^\nu s \tag{1}$$

which is valid if $\Lambda^\mu{}_\nu$ is linked with an $s \in SL(C, 2)$ transformation [covering map $SL(C, 2) \to SO(3, 1)$]. The same linkage is assumed in our model by interpreting s as a rotation (or deformation to ellipsoids) of S^7.

Such an interpretation is possible by considering 8×8 matrices (e.g., a rotation of S^7) as complex 4×4 matrices by introducing the following abbreviations:

$$1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad i = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{2}$$

for 2×2 submatrices composing the real 8×8 matrix (algebra isomorphism).

A special 8-rank covariant antisymmetrical tensor field $\overset{(8)}{\sigma}$ can be defined containing the information of the left- and right-handed Weyl matrices (σ^μ_L and σ^μ_R) and thus of the Dirac matrices γ^μ. By using (1), it can be shown that $\overset{(8)}{\sigma}$ is an invariant tensor field. Considering it as part of the vacuum, it enforces the above-mentioned linkage (exemplified by the cylindrical screw) as is assumed also in (1). $\overset{(8)}{\sigma}$ is a tensorial model for the spin structure of the vacuum.

Special scalar fields

$$\Psi_a(x^\mu) x^a$$

on the 11-dimensional manifold, depending in a linear way on embedding space coordinates x^a of S^7 (vectorial harmonics of S^7) are constructed from ordinary space-time fields $\Psi_a(x^\mu)$ with a spinorial index a, which can be shown to transform as relativistic spinor fields. Postulating for them a field equation of the form

$$\overset{(8)}{\sigma} \wedge d\overset{(1)}{\Psi}$$

leads to a massless Dirac equation

$$\gamma^\alpha \psi_{,\alpha} = 0$$

$\overset{(8)}{\sigma}$ has an additional $U(1)$ internal symmetry (i.e., one not linked to a space-time transformation) which we would like to identify with electromagnetic gauge invariance.

In previous papers (Ebner, 1987, 1988) models for nonrelativistic $SO(3)$ spinors have been found with an S^3 internal space. For the relativistic generalization the use of S^7 was mandatory. It is interesting that the same internal space is favored by people working in grand unification starting from the gauge group $U(1)\times SU(2)\times SU(3)$. Thus, $\overset{(8)}{\sigma}$, besides providing vacuum with a relativistic spin structure and giving space-time a Minkowski-space structure, is also an alternative to the Higgs field.

Other bosonic models for fermions can be found in Finkelstein (1959, 1966, 1968), Williams (1970, 1985), Skyrme (1971), Hasenfratz and 'Hooft (1976), Jackiw and Rebbi (1976), Goldhaber (1976), Friedman and Sorkin (1980), Ringwood and Woodward (1881), Wilczek and Zee (1983), Wu (1984), Carlip (1986), and Benn and Tucker (1983).

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